2) a.

An = 4an-1 – 4an-2 a0 = 1a1 = 4

A2 = 4(4) – 4(1) = 12

A3 = 4(12) – 4(4) = 32

A4 = 4(32) – 4(12) = 80

A5 = 4(80) – 4(32) = 192

A6 = 4(192) – 4(80) = 448

b.

an = 2n \*(n + 1)

c.

Base Case

A1 = 21 \*(1+ 1) = 4

A2 = 22 \*(2+ 1) = 12

A3 = 23 \*(3+ 1) = 32

.

.

An = 2n \* (n + 1)

Inductive Hypothesis

For any number k, Ak can be found by 2k \* (k + 1).

Ak = 2k \*(k+ 1)

Ak + 1 = 2k + 1 \*((k+1)+ 1) =

Ak + 1 = 2k + 1 \*(k+ 2)

Where

A1 = 21 \*(1+ 1) = 4

And A1 + 1 = 21 + 1 \*((1+1)+ 1) 🡪

A2 = 22 \*(2+ 1) = 12

The next term in the sequence can be found by incrementing the value of k. This proves that the next value can be found by the same equation if we keep incrementing the value of k. This is true for all k.

Inductive Conclusion: The equation An = 2n \* (n + 1) is true for all n.

3) a. Among n+1 arbitrarily chosen integers there must exist 2 integers whose difference is divisible by n.

When dividing by n there are only n possible remainders. So, if any of the two arbitrary numbers when divided by n share the same remainder then their difference is divisible by n. This is bound to happen when we are choosing n + 1 arbitrary numbers. Each of the first n numbers could have their own unique remainder when divided by n, but by the pigeonhole principle the n + 1st arbitrary number has to share the same remainder of any of the other n numbers since there are only n remainders available in the first place. Even if all the n arbitrary numbers have their own unique remainder when dived by n the n + 1st number will have the same remainder of one of those other numbers since there aren’t any other unique remainders available. Therefore, by the pigeonhole principle any two numbers from n + 1 numbers will have the same remainder and have a difference that is divisible by n.

4) a. For a relation to be an equivalence relation it needs to satisfy three conditions. Each element needs to be reflexive in its own relation meaning it has to be related to itself. A relation should be symmetric where if there is an arrow pointed toward element b from element a then there should also be an arrow pointed to element a from element b. In other words, element a has to be related to element b the same way element b is related to element a to satisfy symmetry. The last condition is that if element a is related to element b and element b is related to element c, then element a is also related to element c to satisfy the transitive property of equivalence relations. If all three of these conditions are satisfied in a relation than it can be considered an equivalence relation.

b. Equivalence relations don’t have to be mutually disjoint. Equivalence relations can have the same elements repeating in different sets as opposed to partitions. In Equivalence relations, an element may appear in more than one set as it is part of more than one relation or it may appear more than once in the same relation.

For example the equivalence relation: {(a, b) (a, c)}, {(b, a) (b, c)}

Element a can be part of the same equivalence relation set where the element a repeats. The element a can also be repeated in another equivalence relation with a different relation than the set it was in previously. However, in a partition, each element can only be part of one set and there can be no repeats of the elements.

For example: {1, 2, 3, 4, 5, 6}

Can have the partition: {(1, 6) (2, 4) (3, 5)}

Where no elements repeat and are only present in one set making all the sets mutually disjoint.

c. There would be 10! Ways to arrange the beads.

Bead Slot A B C D E F G H I J

10 9 8 7 6 5 4 3 2 1

A bead can be placed for each of the slots A – J. As seen above there are 10 possible beads we can choose from for the first slot, 9 possible beads for the next, and so on. This is because we have exactly 10 distinguishable beads and each of them can only be used once. Once a bead is used for a slot we have one less possible beads to select from for the next slot. Due to the product principle the total number of combination can be counted as 10! = 10 \* 9 \* 8 \* 7 \* 6 \* 5 \* 4 \* 3 \* 2 \* 1 = 3, 628, 800 different combinations.

d. All Beads: A B C D E F G H I J

There are 362, 880 combinations that start with Bead A. This falls under the Equivalence relation A.

There are 362, 880 combinations that start with Bead B. This falls under the Equivalence relation B.

There are 362, 880 combinations that start with Bead C. This falls under the Equivalence relation C.

There are 362, 880 combinations that start with Bead D. This falls under the Equivalence relation D.

There are 362, 880 combinations that start with Bead E. This falls under the Equivalence relation E.

There are 362, 880 combinations that start with Bead F. This falls under the Equivalence relation F.

There are 362, 880 combinations that start with Bead G. This falls under the Equivalence relation G.

There are 362, 880 combinations that start with Bead H. This falls under the Equivalence relation H.

There are 362, 880 combinations that start with Bead I. This falls under the Equivalence relation I.

There are 362, 880 combinations that start with Bead J. This falls under the Equivalence relation J.

Each class consists of 362, 880 different combinations of the beads. The same number of combinations are present for all equivalence classes because they all follow the same combinations with some minor substitutions.

6. a. 210 = 1024

There are 1024 different sequences when flipping a coin ten times. There are two possible choices for each coin flip where the coin can land in either heads or tails. This results in computing

210 = 2 \* 2 \* 2 \* 2 \* 2 \* 2 \* 2 \* 2 \* 2 \* 2 = 1024.

b. (10 \* 9 \* 8 \* 7) / (4 \* 3 \* 2 \* 1) = 5040/24 = 210

There are 210 possible sequences where there are exactly 4 heads. This is calculated by taking into account the location of the four heads in the sequence. The “10 \* 9 \* 8 \* 7” calculates how many possible sequences are possible for the four heads out of the ten flips. The first heads can be any of the ten flips, the second heads can be any of the remaining nine flips, the third heads can be any of the remaining eight flips, and the fourth heads can be any of the remaining eight flips. So, 10 \* 9 \* 8 \* 7 gives us the total number of sequences with exactly four heads. Because the heads in any flip is indistinguishable from the other, order doesn’t matter. So, we can get rid repeating sequences by dividing by (4 \* 3 \* 2 \* 1).

c. (n!)/((k!)(n-k)!) would give us the number of sequence.

The n! on the numerator is the total number of possibilities. In the denominator (k!)(n-k)! accounts for the repeats that shouldn’t occur. The (n-k)! gets rid of the number of possibilities computed for the total and cancels out part of the factorial on the numerator. The k! in the denominator accounts for the case that the order of the elements doesn’t matter.

7. a. There are a total of 16 people consisting of scientists, 5 mathematicians, and 3 geologists. Out of the 16 only five can visit the oil rig. So we have: 16 Choose 5

C (16, 5)

= (16!)/((5!)(16 – 5)!)

=(16!)/((5!)(11!))

= 4, 368

There are 4,368 ways where only 5 of the 16 people can visit the oil rig.

b.

Group can consist of 3 mathematicians: 5 choose 3

Group can consist of 2 geologists: 3 choose 2

Now the total number of combinations can be counted as:

(5C3)\*(3C2)

= ((5!)/((3!)(5-3)!))\* ((3!)/((2!)(3-2)!))

= ((5!)/((3!)(2)!))\* ((3!)/((2!)(1)!))

= 10 \* 3

= 30

When we can only have 3 mathematicians and 2 geologists in our group of 5, we have a total number of 30 different groups.

c.

8 scientists

5 mathematicians

3 geologists

Mathematician can also fill the role of a geologist

(5 Choose 3)(3 Choose 2) + (5 Choose 2)(2 Choose 2)(1 choose 1)

On the left side we have all the possibilities where the mathematician is the mathematician is going as a mathematician or not going at all. On the right we have the possibility where the on mathematician is always going as a geologist.

8. #146 from notes

Base Case:

S(1) = A graph with 1 edge has total degree of 2

S(2) = A graph with 2 edges has total degree of 4

S(3) = A graph with 3 edges has total degree of 6

S(n) = A graph with n edges has a total degree of 2n

Inductive Hypothesis:

Assume S(k) is true in that for a graph with k edges the total degree is 2k

By assumption any graph with k edges has a total degree of 2k. Adding one more edge can be accomplished by either connecting two vertices or adding a loop. Either of these methods adds 2 to the total degree of the graph.

So, the total degree of a graph with k +1 edges has a degree of 2k + 2

2k + 2 = 2 (k+1)

Conclusion

By the principle of mathematical induction S(n) is true for all n

9. #150 from notes

a. In an acquaintance graph any edge consists of a person and any numbers of edges sharing the same vertex would define those two edges or people to be acquainted. In other words, if two edges share the same vertex they are acquainted. Sharing the vertex also increases the degree of the vertex. The higher the degree of the vertex the more edges are sharing the vertex meaning there will be more acquaintances.

b. base case



S(3): In this acquaintance graph there is a vertex that has the total degree of 3. There are 3 people who are acquainted. Furthermore, any individual vertex of the three is acquainted with exactly 2 other people.



S(5): In this acquaintance graph we still have at least one person that is acquainted with an even number of people. That is one of the vertices has a total degree that is even.

Inductive Hypothesis

For any graph S(2k+1) there is at least one vertex with a total degree that is even.

Given that the graphs are connected there is going to be at least one edge whose vertices are always connected to at least one other edge. Each vertex of the edge should be connected to at least one other edge. With these two vertices it shows that the person is acquainted with some even number of people. Taking away one person from the graph there will be an even number of people. That one person can be acquainted to all these people.

Conclusion

It is true that in any graph S(2n+1) there is one person that is acquainted with an even number of other people.

10. #159 from notes

Base Case:

S(1) = A Tree with one vertex has 1-1 = 0 edges

S(2) = A tree with two vertices has 2-1 = 1 edges

S(3) = A tree with three vertices has 3-1 = 2 edges

S(n) = A tree with n vertices has n-1 edges

Inductive Hypothesis:

Assume S(K) for some K >2

Consider a tree with k + 1 vertices. By the laws of graph theory any such graph has a vertex with degree of 1. If I remove a vertex then I’ll have k vertices so should in correlation also have one less edge since there is either no end vertex or start vertex for this edge. This implies that the tree with k + 1 vertices has k edges.

So, S(k+1) is true

Conclusion

So by the principle of Mathematical Induction S(n) is true of all n >= 1